$\downarrow E \qquad \uparrow(1-\alpha_{1L})(1-\alpha_{2L})x \qquad \uparrow(1-\alpha_L)y \qquad \uparrow z$

$\downarrow(1-\alpha_{1S})E \qquad \uparrow(1-\alpha_{2L})x \qquad \uparrow y \qquad \downarrow z$

$\downarrow(1-\alpha_{2S})(1-\alpha_{1S})E \qquad \uparrow x \qquad \downarrow y \qquad \downarrow(1-\alpha_L)z$

$$\text{if: }\left\{\begin{array}{l} \alpha_{1L} = \alpha_{2L} = \alpha_L \\ \alpha_{2S} = \alpha_{1S} = \alpha_S \\ 1-\alpha_S = A \\ 1-\alpha_L = B \end{array}\right\} \text{ we can redraw above model}$$

$\downarrow E \quad \uparrow B^2x \quad \uparrow By \quad \uparrow z$ $\qquad$ ① $E = B^2x + By + z$

$\downarrow AE \quad \uparrow Bx \quad \uparrow y \quad \downarrow z$ $\qquad$ ② $AE = Bx + y - z$

$\downarrow A^2E \quad \uparrow x \quad \downarrow y \quad \downarrow Bz$ $\qquad$ ③ $A^2E = x - y - Bz$

solve ① for z

④ $z = E - B^2x - By$

substitute ④ into ② + ③

$$AE = Bx + y - E + B^2x + By \quad \Big/ \quad A^2E = x - y - BE + B^3x + B^2y$$

simplify

⑤ $E(A+1) = x(B^2+B) + y(B+1)$ $\Big/$ ⑥ $E(A^2+B) = x(B^3+1) + y(B^2-1)$

solve ⑤ for y and plug into ⑥

$$y = \frac{E(A+1) - x(B^2+B)}{B+1} \Rightarrow$$

$$E(A^2+B) = x(B^3+1) + (B^2-1)\left[\frac{E(A+1) - x(B^2+B)}{B+1}\right]$$

$$E(A^2+B) = x(B^3+1) + (B^2-1)\left[\frac{E(A+1) - x(B^2+B)}{B+1}\right]$$

$$\left[\frac{E(A+1)}{B+1} - \frac{Bx\cancel{(B+1)}}{\cancel{B+1}}\right]$$

$$E(A^2+B) = x(B^3+1) + (B^2-1)\left[\frac{E(A+1)}{B+1} - Bx\right]$$

$$= x(B^3+1) + (B-1)\cancel{(B+1)}\left[\frac{E(A+1)}{\cancel{B+1}}\right] - (B^2-1)(Bx)$$

$$E(A^2+B) = x(B^3+1) + (B-1)(E(A+1)) - Bx(B^2-1)$$

$$E\left[(A^2+B) - (B-1)(A+1)\right] = x\left[(B^3+1) - (B^3-B)\right]$$

$$E\left[A^2+\cancel{B} - AB + A - \cancel{B} + 1\right] = x\left[\cancel{B^3}+1-\cancel{B^3}+B\right]$$

$$E\left[A^2 - AB + A + 1\right] = x\left[1+B\right]$$

(7) $$x = \frac{E(A^2-AB+A+1)}{(B+1)}$$

Substitute (7) into (5)

$$y = \frac{E(A+1) - B^2+B\left[\frac{E(A^2-AB+A+1)}{B+1}\right]}{B+1}$$

$$y = E\frac{A+1}{B+1} - \frac{B\cancel{(B+1)}}{\cancel{(B+1)}}\left[\frac{E(A^2-AB+A+1)}{(B+1)}\right]$$

$$y = E\,\frac{A+1}{B+1} - \frac{EB}{B+1}(A^2 - AB + A + 1)$$

(8) $$y = E\left[\frac{A+1-BA^2+B^2A-AB-B}{(B+1)}\right]$$

place (7) and (8) into (1) and solve for z

rewrite (1)

(1) $$E = B^2x + By + z$$

$$E = B^2\left[\frac{E(A^2-AB+A+1)}{B+1}\right] + B\left[\frac{E(A+1-BA^2+B^2A-BA-B)}{(B+1)}\right] + z$$

$$z = \frac{E(B+1)}{(B+1)} - \frac{EB^2(A^2-AB+A+1)}{(B+1)} - \frac{EB(A+1-BA^2+B^2A-BA-B)}{(B+1)}$$

$$z = E\left[\frac{\cancel{B}+1-\cancel{B^2A^2}+\cancel{AB^3}-\cancel{AB^2}-\cancel{B^2}-BA-\cancel{B}+\cancel{A^2B^2}-\cancel{B^3A}+\cancel{B^2A}+\cancel{B^2}}{(B+1)}\right]$$

(9) $$z = E\left(\frac{1-BA}{B+1}\right)$$

Let $\psi = \frac{E}{B+1}$ and rewrite (7), (8), & (9)

(7) $x = \psi(A^2 - AB + A + 1)$

(8) $y = \psi(A + 1 - BA^2 + B^2A - AB - B)$

(9) $z = \psi(1 - BA)$

in 6.27(a) $\alpha_L = 0.5 \Rightarrow B = 0.5 \Rightarrow \psi = \frac{E}{1.5}$

$\alpha_S = 0 \Rightarrow A = 1$

$$x = \frac{E}{1.5}(1 - .5 + 1 + 1)$$

$$= \frac{2.5\,E}{1.5} = \frac{5E}{3}$$

$$y = \frac{E}{1.5}(1 + 1 - .5 + .25 - .5 - .5)$$

$$= \frac{.75\,E}{1.5} = \frac{E}{2}$$

$$z = \frac{E}{1.5}(1 - .5)$$

$$= \frac{.5E}{1.5} = \frac{E}{3}$$

with an albedo = 0.3

$$E = (1 - .3)\, S/4 = 241.5$$

$x = 402.5$ $\quad y = 120.75$ $\quad z = 80.5$

$$\sqrt[4]{\frac{x}{\sigma}} = T_E \qquad \sqrt[4]{\frac{2y}{\sigma}} = T_2 \qquad \sqrt[4]{\frac{2z}{\sigma}} = T_1$$

$T_E = 290K$ $\quad T_2 = 255K$ $\quad T_1 = 231K$